M99/510/H(1)M

INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

MARKSCHEME

May 1999

MATHEMATICS

Higher Level

Paper 1

15 pages

1. By the remainder theorem,

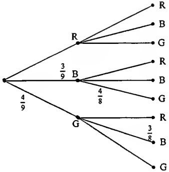
$f(-1) = 6 - 11 - 22 - a + 6$ *(M1)*

$= -20$ *(M1)*

$\Leftrightarrow \quad a = -1$ *(A2)*

Answer: $a = -1$ *(C4)*

2. Using a tree diagram,



$p(\text{BG or GB}) = \left(\frac{3}{9} \times \frac{4}{8}\right) + \left(\frac{4}{9} \times \frac{3}{8}\right)$ *(M1)(M1)*

$= \frac{1}{6} + \frac{1}{6}$ *(A1)*

$= \frac{1}{3}$ *(A1)*

OR $p(\text{BG or GB}) = 2 \times \frac{4}{9} \times \frac{3}{8}$ *(M1)(M1)*

$= \frac{1}{3}$ *(A2)*

Answer: $\frac{1}{3}$ *(C4)*

3. Let a be the first term and d be the common difference, then

$a+d=7$ and $S_4=\frac{4}{2}(2a+3d)=12$ *(M1)*

$\Rightarrow \quad \left.\begin{aligned} a+d&=7 \\ 4a+6d&=12 \end{aligned}\right\}$ *(M1)*

$\Rightarrow \quad a=15, d=-8$ *(A2)*

Answer: $a=15$ *(C2)*
$d=-8$ *(C2)*

4. Substituting gives,

$2(2\lambda+4)+3(-\lambda-2)-(3\lambda+2)=2$ *(M1)*

$\Leftrightarrow \quad 4\lambda+8-3\lambda-6-3\lambda-2=2$ *(M1)*

$\Leftrightarrow \quad -2\lambda=2$

$\lambda=-1$ *(A1)*

Intersection is $(2,-1,-1)$ *(A1)*

Answer: Intersection is $(2,-1,-1)$ *(C4)*

5. $(1-\mathrm{i})z=1-3\mathrm{i}$

$\Leftrightarrow \quad z=\frac{1-3\mathrm{i}}{1-\mathrm{i}}$ *(M1)*

$\Leftrightarrow \quad z=\frac{1-3\mathrm{i}}{1-\mathrm{i}}\times\frac{1+\mathrm{i}}{1+\mathrm{i}}$ *(M1)*

$\Leftrightarrow \quad z=2-\mathrm{i}$ *(A2)*

OR Let $z=x+\mathrm{i}y$

$(1-\mathrm{i})(x+\mathrm{i}y)=1-3\mathrm{i}$ *(M1)*

$x+y-\mathrm{i}(x-y)=1-3\mathrm{i}$

$\left.\begin{aligned} x+y&=1 \\ x-y&=3 \end{aligned}\right\}$ *(M1)*

$\Rightarrow x=2, y=-1$ *(A2)*

Answer: $x=2$ *(C2)*
$y=-1$ *(C2)*

Note: Award *(C4)* for $z=2-\mathrm{i}$

6. The system of equations will not have a unique solution if the determinant of the matrix representing the equations is equal to zero.

Therefore, $\begin{vmatrix} 4 & -1 & 2 \\ 2 & 3 & 0 \\ 1 & -2 & a \end{vmatrix} = 0$ *(M1)*

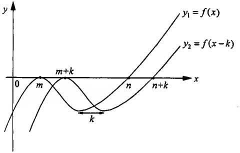
$\Leftrightarrow \quad 4 \times 3a + 2a + 2 \times (-4 - 3) = 0$ *(M1)*

$\Leftrightarrow \quad 14a = 14$ *(M1)*

$a = 1$ *(A1)*

Answer: $a = 1$ *(C4)*

7.



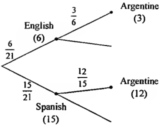
(A2)(A2)

Notes:	The graph of y_2 is y_1 shifted k units to the right. Award *(A2)* for the correct graph. Award *(A1)* for indicating each point of intersection with the x-axis *i.e.* $(m+k, 0)$ and $(n+k, 0)$

Answer: See graph *(C4)*

Note:	Award *(C4)* if the graph of y_2 is drawn correctly and correctly labelled with $m+k$ and $n+k$.

8. Using a tree diagram,



(M2)

Let $p(S)$ be the probability that the pupil speaks Spanish.
Let $p(A)$ be the probability that the pupil is Argentine.

Then, from diagram,

$$p(S|A) = \frac{12}{15}$$ *(A1)*

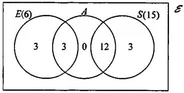
$$= \frac{4}{5}$$ *(A1)*

OR $$p(S|A) = \frac{p(S \cap A)}{p(A)}$$ *(M1)*

$$= \frac{12}{21} \Big/ \frac{15}{21}$$ *(M1)(A1)*

$$= \frac{4}{5}$$ *(A1)*

OR



(M2)

$$p(S|A) = \frac{12}{15}$$ *(A1)*

$$= \frac{4}{5}$$ *(A1)*

Answer: $p(S|A) = \frac{4}{5}$ *(C4)*

9. By implicit differentiation,

$$\frac{d}{dx}(2x^2 - 3y^2 = 2) \Rightarrow 4x - 6y\frac{dy}{dx} = 0 \qquad (M1)$$

$$\Leftrightarrow \quad \frac{dy}{dx} = \frac{4x}{6y} \qquad (A1)$$

When $x = 5$, $50 - 3y^2 = 2$

$\Leftrightarrow \quad y^2 = 16$ *(M1)*

$\Leftrightarrow \quad y = \pm 4$

Therefore $\frac{dy}{dx} = \pm\frac{5}{6}$ *(A1)*

Note: This can be done explicitly

Answers: $\frac{dy}{dx} = \pm\frac{5}{6}$ *(C2)(C2)*

10. (a) A perpendicular vector can be found from the vector product

$$\vec{OP} \times \vec{OQ} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -3 & 2 \\ -2 & 1 & -1 \end{vmatrix} = \vec{i} - 3\vec{j} - 5\vec{k} \qquad (M1)(A1)$$

(b) Area $\Delta OPQ = \frac{1}{2}\left|\vec{OP}\right|\left|\vec{OQ}\right|\sin\theta$, where θ is the angle between $\vec{OP}$ and $\vec{OQ}$ *(M1)*

$$= \frac{1}{2}\left|\vec{OP} \times \vec{OQ}\right|$$

$$= \frac{\sqrt{35}}{2} \qquad (A1)$$

Answers: (a) $\vec{i} - 3\vec{j} - 5\vec{k}$ (or any multiple) *(C2)*

(b) $\frac{\sqrt{35}}{2}$ *(C2)*

11. Given $y = \arccos(1 - 2x^2)$

then $\dfrac{dy}{dx} = \dfrac{-1}{\left(1-(1-2x^2)^2\right)^{1/2}} \times -4x$ *(M1)*

$\dfrac{dy}{dx} = \dfrac{4x}{\left(1-(1-4x^2+4x^4)\right)^{1/2}}$ *(M1)*

$\dfrac{dy}{dx} = \dfrac{4x}{(4x^2-4x^4)^{1/2}}$ *(A2)*

OR

$\cos y = 1 - 2x^2$ *(M1)*

$-\sin y \dfrac{dy}{dx} = -4x$

$\dfrac{dy}{dx} = \dfrac{-4x}{-\sin y} = \dfrac{4x}{\sqrt{1-(1-2x^2)^2}}$ *(M1)*

$\dfrac{dy}{dx} = \dfrac{4x}{\sqrt{4x^2-4x^4}}$ *(A2)*

Answer: $\dfrac{dy}{dx} = \dfrac{4x}{(4x^2-4x^4)^{1/2}}$ **or** $\dfrac{dy}{dx} = \dfrac{4x}{\sqrt{4x^2-4x^4}}$ *(C4)*

12. Let $f(x) = ax^2 + bx + c$ where $a = 1$, $b = (2-k)$ and $c = k^2$. Then for $a > 0$, $f(x) > 0$ for all real values of x if and only if

$b^2 - 4ac < 0$ *(M1)*

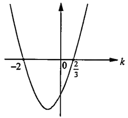
$\Leftrightarrow \ (2-k)^2 - 4k^2 < 0$ *(A1)*

$\Leftrightarrow \ 4 - 4k + k^2 - 4k^2 < 0$

$\Leftrightarrow \ 3k^2 + 4k - 4 > 0$

$\Leftrightarrow \ (3k-2)(k+2) > 0$ *(M1)*

$\Leftrightarrow \ k > \frac{2}{3}, k < -2$ *(A1)*



Answer: $k < -2$, or $k > \frac{2}{3}$ *(C2)(C2)*

13. Let the volume of the solid of revolution be V.

$V = \pi\int_0^a \left((ax+2)^2 - (x^2+2)^2\right)dx$ *(M1)*

$= \pi\int_0^a (a^2x^2 + 4ax + 4 - x^4 - 4x^2 - 4)dx$ *(M1)*

$= \pi\left[\frac{1}{3}a^2x^3 + 2ax^2 - \frac{1}{5}x^5 - \frac{4}{3}x^3\right]_0^a$ *(M1)*

$= \pi\left(\frac{2}{15}a^5 + \frac{2}{3}a^3\right)$ units^3 *(A1)*

$= \frac{2a^3\pi}{15}(a^2+5)$

Note: The last line is not required

Answer: $V = \frac{2a^3\pi}{15}(a^2+5)$ or equivalent *(C4)*

14. Let $u=\frac{1}{2}x+1 \Leftrightarrow x=2(u-1) \Rightarrow \frac{dx}{du}=2$

Then $\int x\left(\frac{1}{2}x+1\right)^{1/2} dx$

$$= \int 2(u-1)\times u^{1/2}\times 2\,du \qquad \textit{(M1)}$$

$$= 4\int\left(u^{3/2}-u^{1/2}\right)du \qquad \textit{(A1)}$$

$$= 4\left[\frac{2}{5}u^{5/2}-\frac{2}{3}u^{3/2}\right]+C \qquad \textit{(M1)}$$

$$= 4\left[\frac{2}{5}\left(\frac{1}{2}x+1\right)^{5/2}-\frac{2}{3}\left(\frac{1}{2}x+1\right)^{3/2}\right]+C \qquad \textit{(A1)}$$

$$= \frac{8}{15}\left(\frac{1}{2}x+1\right)^{3/2}\left(\frac{3}{2}x-2\right)+C$$

Note:	The last line is not required

Answer: $4\left[\frac{2}{5}\left(\frac{1}{2}x+1\right)^{5/2}-\frac{2}{3}\left(\frac{1}{2}x+1\right)^{3/2}\right]+C$ or $\frac{8}{15}\left(\frac{1}{2}x+1\right)^{3/2}\left(\frac{3}{2}x-2\right)+C$ *(C4)*

15. The locus defined by $|z-4-3i|=|z-2+i|$ is the perpendicular bisector of the line joining the points $4+3i$ and $2-i$ in the complex plane. *(M2)*
Correct diagram (see below). *(A2)*

OR

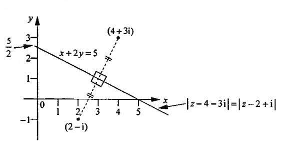
Let $z=x+yi$ then, $|z-4-3i|=|z-2+i|$

$\Leftrightarrow \quad (x-4)^2+(y-3)^2=(x-2)^2+(y+1)^2$ *(M1)*

$\Leftrightarrow \quad -8x-6y+25=-4x+2y+5$

$\Leftrightarrow \quad x+2y=5$ *(M1)*

Therefore the equation of the locus is $x+2y=5$.
Correct diagram (see below). *(A2)*



Note:	Award *(A1)* each for any two of the following (maximum of ***[2 marks]***): Perpendicular line, midpoint $(3, 1)$, y-intercept, x-intercept, gradient.

Answer: Correct locus (see diagram) *(C4)*

16. Given $(1+x)^5(1+ax)^6 = 1+bx+10x^2+\ldots+a^6x^{11}$

$\Leftrightarrow \quad (1+5x+10x^2+\ldots)(1+6ax+15a^2x^2+\ldots) = 1+bx+10x^2+\ldots+a^6x^{11}$ *(M1)*

$\Leftrightarrow \quad 1+(6a+5)x+(15a^2+30a+10)x^2+\ldots = 1+bx+10x^2+\ldots+a^6x^{11}$ *(M1)*

$$\Leftrightarrow \qquad \begin{aligned} 6a+5 &= b & ① \\ 15a^2+30a+10 &= 10 & ② \end{aligned}$$

$$② \quad \begin{aligned} &\Rightarrow & 15a^2+30a &= 0 \\ &\Leftrightarrow & 15a(a+2) &= 0 \\ &\Rightarrow & a &= -2 \end{aligned}$$ *(M1)*

Substitute into ① $\quad b = -7$ *(A1)*

Note: $a \neq 0$ since $a \in \mathbb{Z}^*$

Answers: $a = -2$ *(C2)*
$b = -7$ *(C2)*

17. Let X be the number of counters the player receives in return.

$E(X) = \sum p(x) \times x = 9$ *(M1)*

$\Leftrightarrow \quad \left(\frac{1}{2}\times 4\right)+\left(\frac{1}{5}\times 5\right)+\left(\frac{1}{5}\times 15\right)+\left(\frac{1}{10}\times n\right) = 9$ *(M1)(A1)*

$\Leftrightarrow \quad \frac{1}{10}n = 3$

$\Leftrightarrow \quad n = 30$ *(A1)*

Answer: $n = 30$ *(C4)*

18. Let $\Phi(z) = 0.017$
then $\Phi(-z) = 1 - 0.017 = 0.983$ *(M1)*
$z = -2.12$ *(A1)*

But $z = \frac{x-\mu}{\sigma} = \frac{1-1.02}{\sigma}$ where $x = 1$ kg

Therefore $\frac{1-1.02}{\sigma} = -2.12$ *(M1)*

$\Leftrightarrow \quad \sigma = 0.00943 \text{ kg} = 9.4 \text{ g}$ (to the nearest 0.1 g) *(A1)*

Answer: $\sigma = 9.4$ g (or equivalent) *(C4)*

19. (a) Given $\dfrac{\mathrm{d}v}{\mathrm{d}t} = -kv$

$\Leftrightarrow \quad \int \dfrac{\mathrm{d}v}{v} = -k\int \mathrm{d}t$

$\Leftrightarrow \quad \ln v = -kt + C$ *(M1)*

$\Leftrightarrow \quad v = Ae^{-kt}\ (A = e^{C})$

At $t = 0,\ v = v_0 \Rightarrow A = v_0$

$\Leftrightarrow \quad v = v_0 e^{-kt}$ *(A1)*

(b) Put $v = \dfrac{v_0}{2}$

then $\dfrac{v_0}{2} = v_0 e^{-kt}$ *(M1)*

$\Leftrightarrow \quad \dfrac{1}{2} = e^{-kt}$

$\Leftrightarrow \ln\dfrac{1}{2} = -kt$

$\Leftrightarrow \quad t = \dfrac{\ln 2}{k}$ *(A1)*

Note: Accept equivalent forms, *e.g.* $t = \dfrac{\ln\frac{1}{2}}{-k}$

Answers: (a) $v = v_0 e^{-kt}$ *(C2)*

(b) $t = \dfrac{\ln 2}{k}$ *(C2)*

20. Given $v = \dfrac{(3s+2)}{(2s-1)}$

then acceleration $a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{\mathrm{d}v}{\mathrm{d}s} \times \dfrac{\mathrm{d}s}{\mathrm{d}t} = \dfrac{\mathrm{d}v}{\mathrm{d}s} \times v$ *(M1)*

therefore $\quad a = \dfrac{3(2s-1) - 2(3s+2)}{(2s-1)^2} \times \dfrac{(3s+2)}{(2s-1)}$ *(M1)*

$\Leftrightarrow \quad a = \dfrac{-7(3s+2)}{(2s-1)^3}$ *(M1)*

therefore when $s = 2$, $a = \dfrac{-56}{27}$ *(A1)*

Answer: acceleration $= -\dfrac{56}{27}$ *(C4)*